Approximation of Solutions of Fractional-Order Delayed Cellular Neural Network on $[0, \infty)$

Swati Tyagi, Syed Abbas, Manuel Pinto and Daniel Sepúlveda

Abstract. In this paper, we study a class of fractional-order cellular neural network containing delay. We prove the existence and uniqueness of the equilibrium solution followed by boundedness. Based on the theory of fractional calculus, we approximate the solution of the corresponding neural network model over the interval $[0, \infty)$ using discretization method with piecewise constant arguments and variation of constants formula for fractional differential equations. Furthermore, we conclude that the solution of the fractional-delayed system can be approximated for large $t$ by the solution of the equation with piecewise constant arguments, if the corresponding linear system is exponentially stable. At the end, we give two numerical examples to validate our theoretical findings.

Mathematics Subject Classification. 26A33, 92B20, 34D20, 34K07.

Keywords. Fractional-order, neural network, delay differential equation, approximate solution, error analysis.

1. Introduction

The Fractional-calculus as a generalization of integer order differentiation and integral to non-integer order has been first introduced 300 years ago, though their applications are rather recent. There is a long history of fractional operators which has been already mentioned by Leibnitz in a letter correspondence with L'Hospital in 1695 in [1]. For a long time, the theory of fractional calculus developed as a pure theoretical field has been used by mathematicians only. However, in past few decades, fractional calculus and its applications to various fields such as physics, biology and engineering have attracted attention of many researchers. This generalization of various dynamical equations using fractional derivative has proved to be useful in precise mathematical modeling of several real-world phenomena that arise from various interdisciplinary areas. These fields include control theory [2], viscoelasticity [3], signal processing [4] and social sciences [5]. For more details, we refer to [6–13].